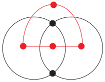
2.1 Finding the Dual Graph

Dual Graph Based Layout: Key stages

- Create an initial dual graph from the abstract description
- Find a planar sub-graph of the initial graph
- Apply transformations to the initial graph to ensure it is connected and to alter the properties possessed by the to-be-embedded Euler diagram
- Improve the layout of the final dual graph
- Form the Euler graph and, hence, the Euler diagram from the final dual graph; in general, the Euler graph is a subgraph of a dual formed from the dual graph
- Use multi-criteria methods to improve the layout of the Euler diagram



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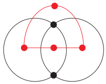


2.1 Finding the Dual Graph

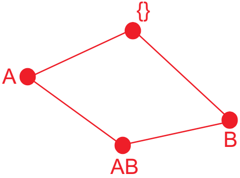
Task 1

Turn an abstract description into a superdual (initial dual graph).

Description

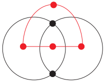
A B AB

Graph



Remark

Create one superdual vertex for each zone description (including {})
Join vertices where node labels have exactly one letter in their symmetric difference.

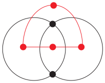


2.1 Finding the Dual Graph: Problems

Task
Find the superduals for each of the following descriptions

1. a
2. a b c
3. a b c ab ac
4. a b ac acd
5. a b c ab ac bc abc
6. a b abc (has a disconnected superdual)
7. a b c d ab ac ad bc bd cd abc abd acd bcd abcd (has a non-planar superdual)

We will use these standard examples again in the rest of the tutorial.



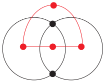
2.1 Finding the Dual Graph: Planarizing

A reduced superdual has the same vertices as the superdual but is planar.

Problem Find a reduced superdual for the last abstract description on the previous slide. Remove as few edges as possible.

Problem Can you find another abstract description that has a non-planar superdual? Try to find one and then find a reduced superdual.

There are well-known algorithms for drawing plane embeddings of a planar graph.



2.1 Transforming the Dual Graph

Properties of the dual graph impact the properties possessed by the embedded diagram.
The diagram is formed by creating the Euler graph from the dual graph.
The dual graph must have a vertex labelled {} next to the infinite face.

Problems

1. For each of the connected, planar superduals from the problems given above, construct an Euler graph. What properties (e.g. no concurrency) are possessed by the Euler diagrams generated?
2. Draw one or two of these superduals without {} next to the infinite face. What, if anything, do you notice about the zones in which the vertices lie when creating the diagram?
3. Try adding edges to, or removing edges from, the superduals, whilst maintaining connectivity and planarity. What impact, if any, do your changes have on the properties possessed by the generated Euler diagrams?

2.1 Transforming the Dual Graph: Concurrent Edge Addition

Definition

A concurrent edge is an edge between two vertices whose labels have a symmetric difference containing more than one curve label.

Task

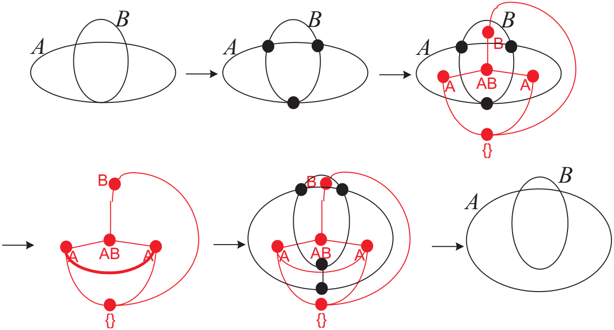
1. For each of the problems given above (except the first), add one or two concurrent edges to the reduced superduals (in most cases, this is the superdual).
2. Then, construct the Euler graph and compare with those produced previously.
3. Have the properties possessed by the diagrams changed?

2.1 Transforming the Dual Graph: Brushing points

Remark

Brushing points can be removed by adding edges to the dual graph.

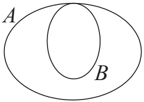
Example

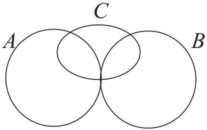


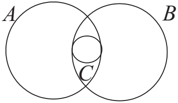
2.1 Transforming the Dual Graph: Brushing points

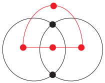
Task

Each of the Euler diagrams below contain a brushing point. Convert each of them to an Euler graph and construct their dual graph. Add edges to the dual graph in order to remove the brushing point(s). If there are choices of edges to add, try the different options to see the impact on the Euler diagram.





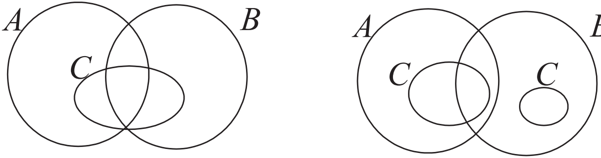




2.1 Transforming the Dual Graph: Triple points

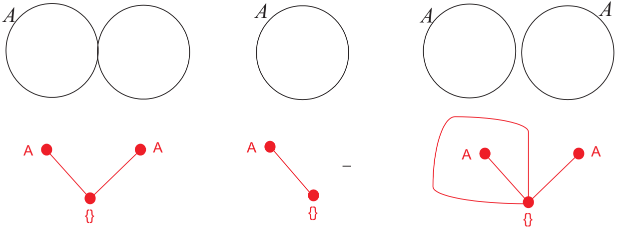
Remark Triple points can be removed by adding edges to the dual graph, as for brushing points, but sometimes more complex sequences of steps are required.

Find a sequence of transformations on the dual graph of the left-hand diagram to yield a dual graph for the right-hand diagram. You may wish to start by drawing both dual graphs.



2.1 Transforming the Dual Graph: Non-simple curves

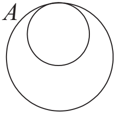
Example

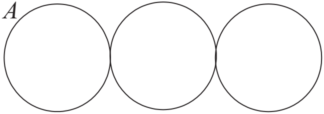


Non-simplicity can be removed in a number of ways.
We have used (a) vertex deletion and (b) edge addition.

2.1 Transforming the Dual Graph: Non-simple

Task Each of the Euler diagrams below contain a non-simple curve. Apply some transformation to the dual graph to remove the non-simple points.

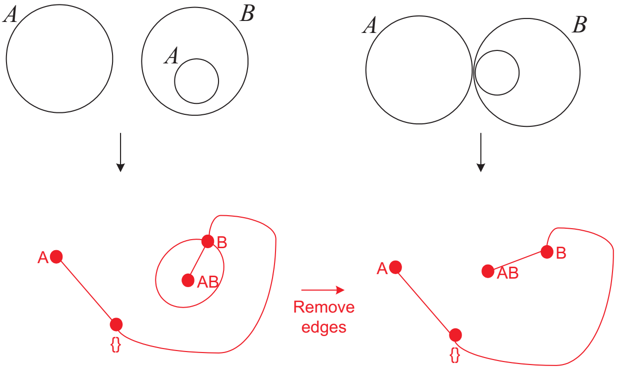




2.1 Transforming the Dual Graph: Non-unique labels

Remark We can reduce the number of times a label is used.

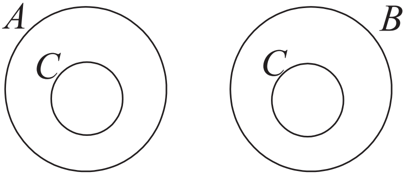
Example



2.1 Transforming the Dual Graph: Non-unique labels

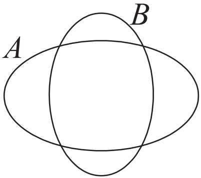
Problem

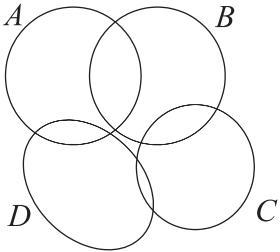
Given the diagram below, find a transformation of the dual graph to remove the duplicated curve label.

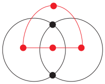


2.1 Transforming the Dual Graph: Disconnected Zones

Task Each of the Euler diagrams below contain a disconnected zone(s). Apply some transformation to the dual graph to remove the disconnected zone(s).







2.1: Summary

- We create a superdual from the abstract description
- This needs to be transformed into a connected, planar graph
- Applying transformations to this graph impacts the properties possessed by the Euler diagram
- Identifying the best transformations is hard!